ECON 440: PUBLIC ECONOMICS
FALL 2011
Assignment #2

Due date: December 1st, 11:30 am in class.

1. Let G be the number of hours of public television broadcast each day. Consider three individuals with preferences:

$$U^A = \frac{G}{4}, \quad U^B = 2 - G^{1/4}, \quad U^C = G - \frac{G^2}{2}.$$

   a) Show that the three consumers have single-peaked preferences.

   b) If the government is choosing G from the range $0 \le G \le 2$, what is the majority voting outcome?

   c) Does this outcome maximize the sum of utilities $W = U^A + U^B + U^C$?

   d) How are the answers to parts a through c altered if the preferences of C becomes $U^C = \frac{G^2}{2} - G$?

2. Three firms have applied for the franchise to operate the cable TV system during the coming year. The annual cost of operating the system is $250 and the demand curve for its services is $P = 500 - Q$, where P is the price per subscriber per year and Q is the expected number of subscribers.

   The franchise is assigned for only one year, and it allows the firm with the franchise to charge whatever price it chooses. The government will choose the applicant according to its lobbying effort. Higher lobbying efforts increases the probability of getting the rent but does not ensure a win. If firm i spends the amount x_i on lobbying activity, it will get the franchise with probability

$$p_i = \frac{x_i}{\sum_{j=1}^{3} x_j}.$$

   [*hint: The winner will set the monopoly price for the service.*]

   a) What is the optimal spending of firm i in response to the total spending of the two other firms $x_{-i} = \sum_{j \neq i} x_j$? Draw the best-response function of firm i to x_{-1}.

   b) Suppose a symmetric equilibrium in lobbying where $x_i = x^*$ for all i. How much will each firm will spend on lobbying?

   c) How does your answer change if there are N extra firms competing for the franchise (assuming again that all firms are identical)?

3. Let a representative consumer have the utility function $U = x_1^{\rho_1} + x_2^{\rho_2} - l$. Remember that $q_i = p_i + t_i$ for good i, and that wage for labor is w.

   a) Find the uncompensated demands $x_i(q_1, q_2, w)$ for each good.

   b) Letting $p_1 = p_2 = 1$, use the inverse elasticity rule to show that the optimal tax rates are related by
   $$\frac{1}{t_2} = \left[\frac{\rho_2 - \rho_1}{1 - \rho_2}\right] + \left[\frac{1 - \rho_1}{1 - \rho_2}\right] \frac{1}{t_1}.$$

   c) Setting $w = 100$, $\rho_1 = 0.75$, and $\rho_2 = 0.5$, find the tax rates required to achieve revenue of $R = 0.5$ and $R = 10$.

   d) Calculate the proportional reduction in demand for the two goods comparing the no-tax position with the position after the imposition of the optimal taxes for both revenue levels. Comments on the results (No points for this part, if I have no comments !).

4. Consider an economy with two consumers of skill levels s_1 and s_2, $s_2 > s_1$. Denote the allocation of the low-skill consumer by x_1, z_1 and that to the high-skill consumer by x_2, z_2. Here x_i is the consumption good and z_i is before tax income.

   a) For the utility function $U = u(x) - \frac{z}{s}$ show that incentive compatibility requires that $z_2 = z_1 + [u(x_2) - u(x_1)]$.

   b) For the utilitarian social welfare function
   $$W = u(x_1) - \frac{z_1}{s_1} + u(x_2) + \frac{z_2}{s_2}.$$
   express W as a function of x_1 and x_2 alone.

   c) Assuming $u(x_h) = \ln x_h$, derive the optimal values of x_1 and x_2 and hence of z_1 and z_2.

   d) Calculate the marginal rate of subsitution for the two consumers at the optimal allocation. Comment on your results. (No comments, no points!)